647510 **MSC4MatC404x** Seat No :________

M.Sc.(Maths.) Semester - 4 (*CBCS*) Examination
April/May-2023 (NEW COURSE)
Graph Theory (Core)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1(A) Answer the following questions. (Any Two) **(10)**

1. Let $G = (V, E)$ be a graph. Prove that G is disconnected graph if and only if there exist disjoint nonempty subsets V_1 and V_2 of V such that: $V = V_1 \cup V_2$ and there is no edge $e \in E$ such that one end vertex of e lies in V_1 and the other lies in V_2.
2. For a simple graph $G = (V, E)$ on n vertices, q edges and k components, show that $q \leq \frac{1}{2}(n-k)(n-k+1)$.
3. Show that in a given transport network G, the value of flow w from the source s to sink t is less than or equal to the capacity of any cut separating s from t.

Q.1(B) Answer the following question. (Any One) **(04)**

1. Define the following terms:
   (I) Degree of a Vertex **(II)** Loop
   (III) Null Graph **(IV)** Regular Graph
2. Explain Maximal Flow in brief.

Q.2(A) Answer the following questions. (Any Two) **(10)**

1. Let $G = (V, E)$ be a connected graph. Show that G is an Euler Graph if and only if degree of each vertex is even.
2. Show that a connected graph G is an Euler graph if and only if G can be decomposed into edge disjoint cycles.
3. State and prove Bondy & Chavatal's Theorem.

Q.2(B) Answer the following question. (Any One) **(04)**

1. Define the following terms:
   (I) Euler Graph
   (II) Unicursal Graph
   (III) Hamiltonian Cycle
   (IV) Closure of a Graph
2. Explain Transport Network with example.

Q.3(A) **Answer the following questions. (Any Two)** **(10)**

1. Let T=(V,E) be a tree with n vertices. Show that $|E(T)| = n - 1$.
2. Let $G = (V, E)$ be a connected graph and S is a cut set for G. If F is a cycle in G, then show that $|E(F) \cap S|$ is even.
3. Let u and v be two distinct vertices of a tree T. Show that there exists a unique path between u and v in T.

Q.3(B) **Answer the following question. (Any One)** **(04)**

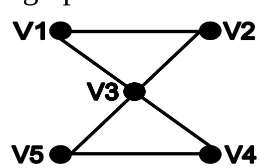
1. Define the following terms:
   (I) Tree **(II)** Pendant Vertex
   (III) Acyclic Graph **(IV)** Minimally Connected Graph
2. Show that if G is an acyclic graph with n vertices and k components, then $|E(G)| = n - k$.

Q.4(A) **Answer the following questions. (Any Two)** **(10)**

1. Show that a connected planner graph G with n vertices, e edges has precisely $f = e - n + 2$ regions.
2. Prove that Kuratowaski's first graph and second graph, both are non-planer graphs.
3. Let $G = (V, E)$ be a simple connected graph with n vertices, e edges and f faces. Then show that: **(I)** $e \geq \frac{3f}{2}$ **(II)** $e \leq 3n - 6$

Q.4(B) **Answer the following question. (Any One)** **(04)**

1. Define Adjacency Matrix with an example.
2. Find tree sub-graphs for the following graph



Q.5(A) **Answer the following questions. (Any Two)** **(10)**

1. Prove that every tree with at least two vertices is always 2-chromatic graph.
2. Prove that the vertices of every planer graph can be properly colored with five colors.
3. Show that a graph of n vertices is a complete graph if and only if its chromatic polynomial is $P_n(\lambda) = \lambda(\lambda - 1)(\lambda - 2) \cdots (\lambda - n + 1)$.

Q.5(B) **Answer the following question. (Any One)** **(04)**

1. Explain coloring of a graph with example.
2. Show that a graph with at least one edge is 2-chromatic if and only if it has no circuits of odd length.

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